

On the lagrangian of N=1, D=10 dual supergravity

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Abstract

It is shown how one can construct the lagrangian of dual supergravity by means of the equations of motion derived from the superspace approach.

Dual N=1, D=10 supergravity is equivalent to the usual N=1, D=10 supergravity at the level of the minimal lagrangian (i.e. one which contains not more than 2-nd derivatives). But the condition of anomalies cancellation implies that some nonminimal terms must be added to the lagrangian. The anomalies can be taken into account in the superspace approach. It appears (see [1, 2]), that in the usual supergravity the nonminimal corrections turn out to be the infinite series of terms each of them should be calculated by the perturbation theory in the string coupling constant. Meanwhile the dual supergravity lagrangian has a finite number of terms which can be written explicitly [3]. But the superspace approach gives us only the equations of motion; the building of the lagrangian is a matter of some difficulty. Initially the dual supergravity lagrangian has been obtained in [4] by means of the dual transformation from the usual supergravity lagrangian. Some partial results on the nonminimal corrections have been obtained in [2, 5] but even the bosonic part of the lagrangian has not been found completely. The main difficulty is to transform the equations of motions to the form where they immediately follow from the lagrangian. The presence of additional constraints (see below) also complicates the situation. Our purpose here is to formulate a method, which helps to resolve the difficulties. In this paper the minimal lagrangian of N=1, D=10 dual supergravity is constructed by means of the equations of motion, derived from the superspace approach. We hope that this work helps us to build the anomaly free lagrangian when the nonminimal corrections to the equations of motion will be written explicitly.

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The multiplet of the dual supergravity contains: $e_m{}^a$ – graviton, $\psi_m{}^\alpha$ – gravitino, ϕ – dilaton, χ_α – dilatino and $M_{m_1\dots m_6}$ – antisymmetrical potential (we do not consider the contribution of the gauge matter here). The equations of motion for these fields depend on the constraints imposed on the components of the correspondent superfields. We use the constraints introduced in [6] which are supposed to be the simplest ones. Equations of motion in this system of constraints (after some fields redefinition) have been obtained in [3] and are listed in Appendix A of this paper. These equations have the important property: they are linear in fields ϕ and χ . Hence the lagrangian must be linear in dilaton and dilatino fields:

$$L = (\phi S_1 + \chi_\alpha S_2^\alpha) | \quad (1)$$

(the vertical line denotes the 0-component of a superfield).

In order to find S_1 and S_2^α it is necessary to use the torsion Bianchi identities. In this set of constraints they do not fix the curvature \mathcal{R}_{abcd} and the torsion T_{abc} and $T_{ab}{}^\alpha$ components. The general expressions for S_1 and S_2^α through these components, which contain not more than the 2-nd derivatives, are:

$$S_1 = \mathcal{R} + a T^2, \quad S_2 = b \Gamma^{ab} T_{ab}, \quad (2)$$

where $\mathcal{R} = \mathcal{R}_{ab}{}^{ab}$, $T^2 = T_{abc} T^{abc}$, the T_{ab} denotes $T_{ab}{}^\alpha$; the common numerical factor before the lagrangian is not important here. The quantities S_1 and S_2 must vanish on shell. But the torsion Bianchi identities guarantee, that the following relations are valid on shell (see [3])

$$\mathcal{R} - \frac{1}{3} T^2 = 0 \quad (3)$$

$$\Gamma^{ab} T_{ab} = 0 \quad (4)$$

(in the set of constraints presented in Appendix A; however the analogous relations take place in some other systems of constraints, for instance in [1, 7]). Hence $a = -1/3$.

In order to find factor b it is necessary to consider some equation for $e_m{}^a$, ψ_m or $M_{m_1\dots m_6}$. For this purpose we must write down the 0-components (see Appendix B) of the relations (A.3) – (A.7) and transform them to the form where they may be derived from the lagrangian immediately.

We demonstrate, how one can do it for the potential $M_{m_1\dots m_6}$. To write down the 0-component of the relation (A.5) let us express the derivative with the flat index D_a through the derivative with the world index D_m :

$$D_a| = e_a^m D_m| - \frac{1}{2} \psi_a^\alpha D_\alpha| \quad (5)$$

We take the spinor derivative $D_\alpha T_{abc}$ from the solution of the torsion Bianchi identities (see [3]):

$$DT_{abc} = -\frac{1}{2} \Gamma_{abc}^{fh} T_{fh} + \alpha \Gamma_{abc} \Gamma^{fh} T_{fh} \quad (6)$$

(the Bianchi identities give us only on-shell fields values, i.e. while $\Gamma^{fh} T_{fh} = 0$; so one can choose arbitrary α in (6)). Then let us substitute the expressions (B.3) and (B.4) for the connection $\phi_{ma}^b|$ into the derivative D_m defined according to (B.2) and after that substitute the torsion components $T_{ab}^\alpha|$ and $T_{abc}|$ from (B.5) and (B.6).

The terms

$$\beta \psi_{[a} \Gamma_{bc]} (\phi \Gamma^f T_{d]f} - D_{d]} \chi - \frac{1}{36} \Gamma_{d]} \hat{T} \chi - \frac{1}{24} \hat{T} \Gamma_{d]} \chi)| = 0 \quad (7)$$

$$\gamma \psi_{[a} \Gamma_{bcd]} (\hat{D} \chi + \frac{1}{9} \hat{T} \chi)| = 0 \quad (8)$$

are equal zero on shell and thus can be added to resulting expression with arbitrary factors β and γ . However the lagrangian must be invariant relative to the transformation

$$M_{m_1\dots m_6} \rightarrow M_{m_1\dots m_6} + \partial_{[m_1} f_{m_2\dots m_6]} , \quad (9)$$

which does not involve other fields. Consequently it must contain $M_{m_1\dots m_6}$ only through the field-strength $\partial_{[m_1} M_{m_2\dots m_7]}$. So the variation of action with respect to the potential must be the full derivative. This claim fixes all unknown factors unambiguously: $\alpha = 1/2$, $\beta = 3/2$, $\gamma = 0$.

As a result we derive the equation of motion for $M_{m_1\dots m_6}$:

$$(\phi M_{[abc} - \frac{1}{8} \phi \psi_f \Gamma^{[f} \Gamma_{[abc} \Gamma^{h]} \psi_{|h]} + \frac{3}{2} \psi_{[a} \Gamma_{bc]} \chi)_{;d]} = 0 \quad (10)$$

where

$$M_{abc} \equiv \frac{1}{6!} \epsilon_{abc}^{d_1\dots d_7} M_{d_1\dots d_6; d_7} ,$$

the semicolon denotes the ordinary covariant derivative which depends on the ordinary vielbein.

Taking into account (see (B.5), (B.6), (B.8)) that 0-components are

$$\mathcal{R}| = \frac{1}{4} M_{abc}^2 + \dots; \quad T_{abc}| = M_{abc} + \dots; \quad \Gamma^{ab}T_{ab}| = -\frac{1}{8} M_{abc}\Gamma^{ab}\psi^c + \dots;$$

where the dots denote the terms of the lower order in the M-field, it is easy to see that the equation (10) can be derived from the lagrangian (1),(2) with $b = 2$.

So the lagrangian must be the 0-component of the following stuff:

$$\mathcal{L} = \phi (\mathcal{R} - \frac{1}{3} T^2) + 2\chi \Gamma^{ab}T_{ab} \quad (11)$$

But there is an ambiguity in the field representation of the quantity $\mathcal{R}|$ at the off-shell level. Indeed, it is necessary to know the curvature components with the spinor indices in order to calculate $\mathcal{R}|$ from the formula $\mathcal{R}| = E_b^N E_a^M \mathcal{R}_{MN}{}^{ab}|$. We can take it from the solution of the torsion Bianchi identities in [3]. But as mentioned before the Bianchi identities give us only on-shell fields values. Consequently $\mathcal{R}|$ may be presented in different forms which are equivalent up to the term

$$\psi_a \Gamma^a \Gamma^{bc} T_{bc}| , \quad (12)$$

vanishing on-shell. One of the possible variants is presented in Appendix B eq. (B.8). It contains terms with M_{abc} which lead to

$$(\mathcal{R} - \frac{1}{3} T^2)| = -\frac{1}{12} M_{abc}^2 + \frac{1}{48} \psi_a \Gamma^{[a} \hat{M} \Gamma^{b]} \psi_b + \dots$$

where $\hat{M} = M^{abc} \Gamma_{abc}$.

Substituting this expression into (11) one can derive the lagrangian which leads to the true equation for $M_{m_1 \dots m_6}$ (10). But if we added the term (12) to (B.8) we would get a wrong equation, different from (10). So the only expression for $\mathcal{R}|$ suitable for us is the eq. (B.8).

We see that equations (3), (4), (10) fix all the terms in the lagrangian without the equations of motion for $e_m{}^a$ and $\psi_m{}^\alpha$. But the straightforward calculation demonstrates that variations of L with respect to $e_m{}^a$, $\psi_m{}^\alpha$ -fields vanish if one takes into account the equations of motion (A.3) – (A.7),

derived from the solution of Bianchi identities. So all the assumptions about the lagrangian structure are confirmed.

Finally:

$$\begin{aligned}
L = & \phi R - \frac{1}{2} \phi \psi_a \Gamma^{abc} \psi_{c;b} - \frac{1}{2} \phi_{;a} \psi^a \Gamma_b \psi^b + 2 \psi_a \Gamma^{ab} \chi_{;b} + \\
& - \frac{1}{12} \phi M_{abc}^2 + \frac{1}{48} \phi \psi_a \Gamma^{[a} \hat{M} \Gamma^{b]} \psi_b - \frac{1}{4} \chi \Gamma^{ab} \psi^c M_{abc} - \\
& - \frac{1}{3 \cdot 256} \phi (\psi^d \Gamma_{dabc} \psi^f)^2 + \frac{1}{64} \phi (\psi_a \Gamma_b \psi_c)^2 + \frac{1}{32} \phi (\psi^a \Gamma^b \psi^c) (\psi_a \Gamma_c \psi_b) - \\
& - \frac{1}{16} \phi (\psi_a \Gamma^b \psi_b)^2 + \frac{1}{8} (\chi \Gamma_{ab} \psi_c) (\psi^a \Gamma^c \psi^b) - \frac{1}{4} (\chi \Gamma_a \Gamma_b \psi^b) (\psi^a \Gamma_c \psi^c)
\end{aligned} \quad (13)$$

We shall write below the supersymmetry transformations for the fields of the gravity multiplet. They are the variations of the correspondent superfields under the shift $\delta z^M = \varepsilon^M$, $\varepsilon^M| = (0, \varepsilon^\alpha)$ and may be easily obtained for $e_m{}^a$, ψ_m , ϕ and χ . To derive the supersymmetry transformation for the $M_{m_1 \dots m_6}$ it needs to write the 0-component of variation $\delta T_{abc} = \varepsilon^\alpha D_\alpha T_{abc}$, taking $D_\alpha T_{abc}$ from (6) and $T_{abc}|$ from (B.6). Choosing $\alpha = 0$ in (6) one can transform this variation to the form:

$$\epsilon_{n_1 n_2 n_3}{}^{m_1 \dots m_7} (\delta M_{m_1 \dots m_6} + 3 \psi_{m_1} \Gamma_{m_2 \dots m_6} \varepsilon)|_{m_7} = 0 \quad (14)$$

The relation (14) defines $\delta M_{m_1 \dots m_6}$ unambiguously up to the transformation of the form (9).

So the supersymmetry transformations in the system of constraints (A.1) and (A.2) are:

$$\begin{aligned}
\delta e_m{}^a &= \frac{1}{2} \psi_m \Gamma^a \varepsilon \\
\delta \psi_m &= 2 \varepsilon_{;m} - \frac{1}{72} (3 \hat{T} \Gamma_m + \Gamma_m \hat{T})| \varepsilon - \frac{1}{2} C_{mpq} \Gamma^{pq} \varepsilon \\
\delta \phi &= -\chi \varepsilon \\
\delta \chi &= -\frac{1}{2} \phi_{;m} \Gamma^m \varepsilon + \frac{1}{4} (\psi_m \chi) \Gamma^m \varepsilon + \frac{1}{36} \phi \hat{T}| \varepsilon \\
\delta M_{m_1 \dots m_6} &= -3 \psi_{[m_1} \Gamma_{m_2 \dots m_6]} \varepsilon
\end{aligned} \quad (15)$$

where

$$C_{klm} = \frac{1}{8} (2\psi_k \Gamma_{[l} \psi_{m]} + \psi_l \Gamma_k \psi_m) .$$

A lot of work need to be done to check the invariance of lagrangian (13) relative to transformations (15). The following speculations help to reduce it.

Let us divide the supersymmetry transformations by two parts:

$$\delta \int eL = \int e(A + B)\varepsilon \quad (16)$$

where $e = \det e_m{}^a$. The part A contains all the terms which can be written symbolicaly in powers of fields and derivatives:

$$A = (\chi + \phi\psi) \times (R + \partial^2 + M\partial + \psi^2\partial)$$

Terms in B do not contain derivatives:

$$B = (\chi + \phi\psi) \times (M^2 + \psi^2 M + \psi^4)$$

(except of the derivative inside M : here M denotes the field-strength M_{abc} but not the potential).

Straightforward calculations show that $A = 0$. Then it appears that $B = 0$ too.

To prove this let us suppose that fields obey the equations of motion. Then one can express the terms of kind A through the terms of kind B and substitute them in the right-hand side of (16). Note, that the left-hand side of (16) must vanish on-shell then $B = -A = 0$. But the term B does not contain derivatives, therefore it is not changed after this substitution. So $B = 0$ in any case, not only on the mass-shell.

Let us suppose now that some terms with derivatives remain in A because they cannot be expressed through the terms without derivatives by means of the equations of motion. In this case we have a new differential equation $A = 0$. But all the field equations are the equations of motion or their consequences. As a result we came to the contradiction.

So in order to check the invariance of action relative to the supersymmetry transformations it is necessary to show only that $A = 0$. It was made by the author of this paper.

The lagrangian (13) does not contain the kinetic terms of dilaton and dilatino.² However, it is possible to transform the Einstein's term of action ϕR to the canonical form by means of the vielbein conformal transformation. Dilaton kinetic term appears as a result. The dilatino kinetic term could arise if one diagonalize the terms with the derivatives of ψ_m and χ in (13). Indeed, the field change $\{\Phi_i\} \rightarrow \{\Phi'_i\}$

$$\begin{aligned} e_m^a &= e^{\phi'/6} e_m^{a'} \\ \psi_m &= 2 e^{\phi'/12} (\psi_m' - \frac{1}{6\sqrt{2}} \Gamma_m' \chi') \\ \phi &= e^{-4\phi'/3} \\ \chi &= -\frac{2\sqrt{2}}{3} e^{-17\phi'/12} \chi' \\ M_{m_1 \dots m_6} &= 2 M_{m_1 \dots m_6}' \end{aligned} \tag{17}$$

leads to the lagrangian L'

$$\int e L = 4 \int e' L'$$

with the canonical kinetic terms:

$$\begin{aligned} L' &= \frac{1}{4} R' + \frac{1}{2} \phi'^{a'} \phi'_{;a'} - \frac{1}{12} e^{-2\phi'} M_{abc}^2 - \frac{1}{2} \psi_a' \Gamma^{abc} \psi_{c;b'} + \frac{1}{2} \chi' \Gamma^a \chi'_{;a} - \\ &- \frac{1}{\sqrt{2}} \phi'_{;b'} \psi_a' \Gamma^b \Gamma^a \chi' + \frac{1}{24} e^{-\phi'} (\psi_a' \Gamma^{[a} \hat{M}' \Gamma^{b]} \psi_b' + \sqrt{2} \chi' \Gamma^a \hat{M}' \psi_a') + \dots , \end{aligned} \tag{18}$$

where the dots denote the terms of 4-th order in fermionic fields. It is easy to see that the lagrangian (18) is the lagrangian of dual supergravity [4] rewritten in the notations used here.

Note in conclusion that the lagrangian (13) in fields parametrization [3] is simpler than lagrangian [4] in fields parametrization [8]. It allows us to hope that nonminimal terms which must be added for cancellation of anomalies are more simple in this parametrization too. In particular, they must be independent of the ϕ and χ fields, because variations of the action with respect to these fields (A6) and (A7) have no anomalous corrections [3].

²Absence of the dilaton kinetic term was noted in [2] for the lagrangian obtained there.

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Appendix A: constraints and equations of motion

The constraints, the solution of Bianchi identities and the equations of motions are taken from [3]. \mathcal{R}_{abcd} here corresponds to $-R_{abcd}$ in [3]; all other notations are the same. All necessary properties of Γ -matrices one can find in [8].

We use the following constraints for the torsion components:

$$\begin{aligned} T_{\alpha\beta}^c &= \Gamma_{\alpha\beta}^c, \quad T_{\alpha\beta}^\gamma = T_{\alpha b}^c = 0 \\ T_{a\beta}^\gamma &= \frac{1}{72} (\hat{T}\Gamma_a)_\beta^\gamma, \quad \text{where } \hat{T} = T^{abc}\Gamma_{abc}. \end{aligned} \quad (A.1)$$

The Bianchi identities for the 7-form $N = dM$ relate the torsion components T_{abc} with the components of N -field which are the field-strength of the potential $M_{m_1\dots m_6}$:

$$N_{\alpha\beta a_1\dots a_5} = -(\Gamma_{a_1\dots a_5})_{\alpha\beta}, \quad N_{abc} = T_{abc} \quad (A.2)$$

where

$$N_{abc} \equiv \frac{1}{7!} \epsilon_{abc}^{b_1\dots b_7} N_{b_1\dots b_7},$$

all other components of N are equal zero.

Equation of motion for the graviton e_m^a :

$$\phi \mathcal{R}_{ab} + D_{(a} D_{b)} \phi - \frac{1}{36} \phi \eta_{ab} T^2 + T_{c(a} \Gamma^c \Gamma_{b)} \chi = 0 \quad (A.3)$$

Equation of motion for the gravitino ψ_m^α :

$$\phi \Gamma^b T_{ab} - D_a \chi - \frac{1}{36} \Gamma_a \hat{T} \chi - \frac{1}{24} \hat{T} \Gamma_a \chi = 0 \quad (A.4)$$

Equation of motion for the $M_{m_1\dots m_6}$:

$$D_{[a} (\phi T_{bcd]}) + \frac{3}{2} T_{[ab} \Gamma_{cd]} \chi + \frac{3}{2} \phi T_{[abcd]}^2 = 0 \quad (A.5)$$

Together with the equations (3), (4)

$$\mathcal{R} - \frac{1}{3} T^2 = 0 \quad (A.6)$$

$$\Gamma^{ab} T_{ab} = 0 \quad (A.7)$$

they form the complete system of equations of motion defining the fields dynamics.

Although the ϕ and χ enter in the lagrangian in the noncanonical manner, they also obey the wave equations which can be easily derived from (A.3) + (A.6) + (A.7) and (A.4) + (A.7):

$$D^a D_a \phi + \frac{1}{18} \phi T^2 = 0 \quad (A.8)$$

$$\hat{D} \chi + \frac{1}{9} \hat{T} \chi = 0 \quad (A.9)$$

Appendix B: 0-components of superfields

The method of transition from superfields to 0-components is the standard one and has been described in [9]. We present here only the final result. The vertical line denotes a 0-component of a superfield.

The supervielbein is defined as:

$$\begin{aligned} E_m{}^a | &= e_m{}^a, & E_m{}^\alpha | &= \frac{1}{2} \psi_m{}^\alpha \\ E_\mu{}^a | &= 0, & E_\mu{}^\alpha | &= \delta_\mu^\alpha \end{aligned} \quad (B.1)$$

The spin-connection $\phi_{Ma}{}^b$, corresponding to D_M , is defined by

$$D_M V^a = \partial_M V^a + V^b \phi_{Mb}{}^a, \quad (B.2)$$

where $M = (m, \mu)$, V^a – is a vector. We suppose:

$$\phi_{ma}{}^b | = \omega_{ma}{}^b, \quad \phi_{\mu a}{}^b | = 0 \quad (B.3)$$

In the system of constraints (A.1) the spin-connection $\omega_{abc} = e_a{}^m \omega_{mbc}$ takes the form:

$$\omega_{abc} = \omega_{abc}^{(0)} + \frac{1}{2} T_{abc} | + C_{abc}, \quad (B.4)$$

where

$$C_{abc} = \frac{1}{8} (2 \psi_a \Gamma_{[b} \psi_{c]} + \psi_b \Gamma_a \psi_c) ,$$

$\omega_{abc}^{(0)}$ is the ordinary spin-connection depending only on ordinary vielbein e_m^a .

The torsion T_{ab}^α in the system of constraints (A.1) is:

$$T_{ab}| = \psi_{[b;a]} - \frac{1}{144} (\Gamma_{[a} \hat{T} + 3\hat{T}\Gamma_{[a})|\psi_{b]} + \frac{1}{4} \Gamma^{cd} \psi_{[a} C_{b]cd} , \quad (B.5)$$

where the semicolon denotes the ordinary covariant derivative with the spin-connection $\omega_{abc}^{(0)}$.

The relation between the N -field components with world and flat indices (see (A.2)) leads to:

$$T_{abc}| = N_{abc}| = M_{abc} - \frac{1}{8} \psi^d \Gamma_{dabc} \psi^f \quad (B.6)$$

where

$$M_{abc} \equiv \frac{1}{6!} \epsilon_{abc}^{d_1 \dots d_7} M_{d_1 \dots d_6; d_7} , \quad M_{a_1 \dots a_6} \equiv e_{a_1}{}^{m_1} \dots e_{a_6}{}^{m_6} M_{m_1 \dots m_6} .$$

The supercurvature $\mathcal{R}_{mna}{}^b$, corresponding to the spin-connection $\phi_{ma}{}^b$, is:

$$\mathcal{R}_{mna}{}^b = 2 \partial_{[m} \phi_{n]a}{}^b - 2 \phi_{[m|a}{}^c \phi_{n]c}{}^b \quad (B.7)$$

The explicit expression for the 0-component of $\mathcal{R}_{abcd}|$ through the ordinary curvature R_{abcd} , corresponding to the spin-connection $\omega_{abc}^{(0)}$, is very cumbersome. So we present here only the expression for $\mathcal{R}| = \mathcal{R}_{ab}{}^{ab}|$:

$$\begin{aligned} \mathcal{R}| &= R - \frac{1}{2} \psi_a \Gamma^{abc} \psi_{c;b} + \frac{1}{2} (\psi^a \Gamma_b \psi^b)_{;a} + \frac{1}{4} T_{abc}^2 | + \frac{1}{8} \psi_a \Gamma_b \psi_c M^{abc} + \\ &+ \frac{1}{64} (\psi_a \Gamma_b \psi_c)^2 + \frac{1}{32} (\psi_a \Gamma_b \psi_c) (\psi^a \Gamma^c \psi^b) - \frac{1}{16} (\psi_a \Gamma_b \psi^b)^2 \end{aligned} \quad (B.8)$$

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